Algorithmic Aspects of Topology Control Problems for Ad hoc Networks

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Topology Control

- Assign powers to the nodes of an ad hoc network so as to induce a graph with appropriate properties (e.g. induced graph is connected).
- Battery power is a precious resource.
- Two objectives: Minimizing maximum power and minimizing total power.

Power Threshold Values

- Power threshold $p(x, y)$: Minimum transmission power to be assigned to x so that a signal from x can reach y.
- Assumed to be *symmetric*; i.e., for all nodes x and y, $p(x, y) = p(y, x)$.

Graph from Power Assignment

- Model used in [Ramanathan & Rosales-Hain, 2000].
- First, construct the directed graph (using the radio propagation model): Add directed edge (x, y) if the power assigned to x is at least $p(x, y)$.
- For each pair of nodes u and v, if **both** (u, v) and (v, u) are present, then replace them by the undirected edge $\{u, v\}$.
- Delete all of the remaining directed edges.
- Two way communication model.
- The directed graph model can also be used.

Problem Formulation and Notation

- A topology control problem is specified by a triple $\langle \mathbb{M}, \mathbb{P}, \mathbb{O} \rangle$.
  - $\mathbb{M} \in \{\textsc{Dir}, \textsc{Undir}\}$ represents the graph model.
  - $\mathbb{P}$ represents the desired graph property.
  - $\mathbb{O} \in \{\textsc{MaxP}, \textsc{TotP}\}$ represents the minimization objective (Max Power or Total Power).

Example: $\langle \textsc{Undir}, \text{1-NC}, \textsc{MaxP} \rangle$

Interpretation: Assign powers so that resulting undirected graph is connected and the maximum power assigned to nodes is minimized.

Summary of Previous Work

- Minimizing Max Power [Ramanathan & Rosales-Hain, 2000]
  - Model for topology control problems.
  - Algorithms for $\langle$UNDIR, 1-NC, MAXP$\rangle$ and $\langle$UNDIR, 2-NC, MAXP$\rangle$.
  - Heuristics for distributed versions.
- Minimizing Total Power
  - NP-completeness of $\langle$UNDIR, 1-NC, TOTP$\rangle$ [Chen & Huang, 1989].
  - NP-completeness of $\langle$UNDIR, 1-NC, TOTP$\rangle$ for points in 2D space [Clementi et al, 1997].
  - 2-approximation for $\langle$UNDIR, 1-NC, TOTP$\rangle$ [Chen & Huang, 1989, Kirousis et al, 1997].

Main Contributions

- A general approach leading to a polynomial algorithm for minimizing maximum power for **monotone** graph properties.
- NP-completeness results showing the necessity of monotone properties.
- A general approach leading to an approximation algorithm for minimizing the total power for some monotone properties.
  - A new approximation algorithm for the problem of minimizing the total power for inducing a 2-node-connected graph.
- Empirical results showing the performance of the algorithm.

Monotone Graph Property

- Property unaffected by the addition of edges to the induced graph.
- Allows us to increase powers of nodes without affecting the property.

Examples:

- Monotone:
  - Connectedness.
  - Diameter $\leq k$.
- Not monotone:
  - Acyclicity.
  - Max node degree $\leq d$.

Algorithm for Minimizing Maximum Power

Main Ideas:

1. There is an optimal solution in which each node is assigned the same power. (Reason: Monotonicity of $\mathbb{P}$.)
2. For a system with n nodes, the number of candidate solution values is $O(n^2)$.
3. Binary search over the candidate solution values – $O(\log n)$ calls to the algorithm for testing property $\mathbb{P}$ suffice.

Other Results for Minimizing MAXP

- If $\mathbb{P}$ is not monotone, then minimizing maximum power is NP-complete.
- Minimizing the number of nodes with maximum power is NP-complete even for monotone properties.

General Heuristic for Total Power

Assumptions: Property $\mathbb{P}$ monotone and efficiently testable; power thresholds are symmetric.

Outline:

1. Construct complete graph $G_c(V, E_c)$, where each edge $\{x, y\}$ has weight $w(x, y) = p(x, y)$ (power threshold).

2. Construct an edge subgraph $G'(V, E')$ of G_c so that G' satisfies $\mathbb{P}$ and the total weight of edges in E' is minimum or near-minimum.

3. For each node x, power(x) is given by

   Max$\{w(x, y)$: $\{x, y\}$ is an edge in $G'\}$.

Note: Step 2 depends on $\mathbb{P}$.

Examples from the General Outline

Ex 1: Approximating $\langle \textsc{Undir}, 1\text{-NC}, \textsc{TotP} \rangle$

- In Step 2, construct a minimum spanning tree for G_c [Kirousis et al, 1997].

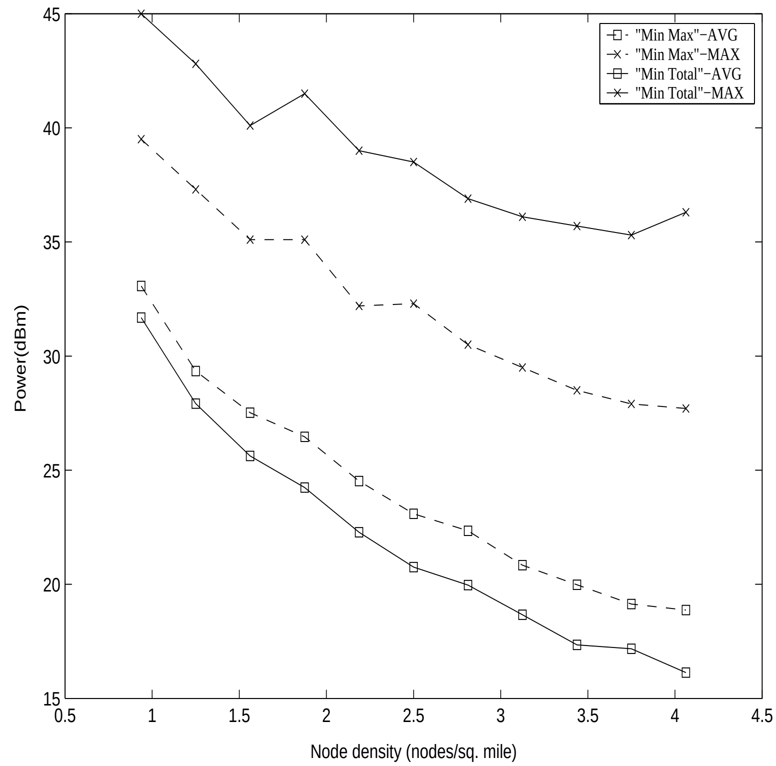
Ex 2: Approximating $\langle \textsc{Undir}, 2\text{-NC}, \textsc{TotP} \rangle$:

- In Step 2, use the heuristic for minimum weight 2-NC subgraph from [Khuller & Raghavachari,1996].

Experimental Results

- Results are for ⟨UNDIR, 2-NC, TOTP⟩.
- Experimental setup similar to that discussed in [Ramanathan & Rosales-Hain, 2000].
- Comparison with the results for ⟨UNDIR, 2-NC, MAXP⟩ from [RR, 2000].
- Both uniform node density and skewed node density were considered.

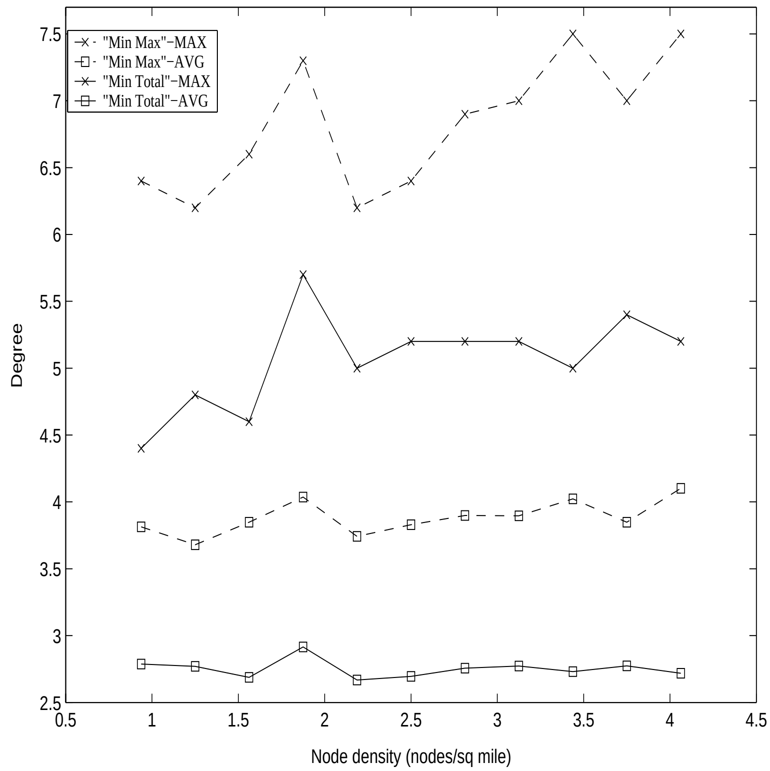
Uniform Node Density



Power vs Node Density

- Average power less than that of [RR, 2000] by 5% to 15%.
- Maximum power is 14% to 31% larger.

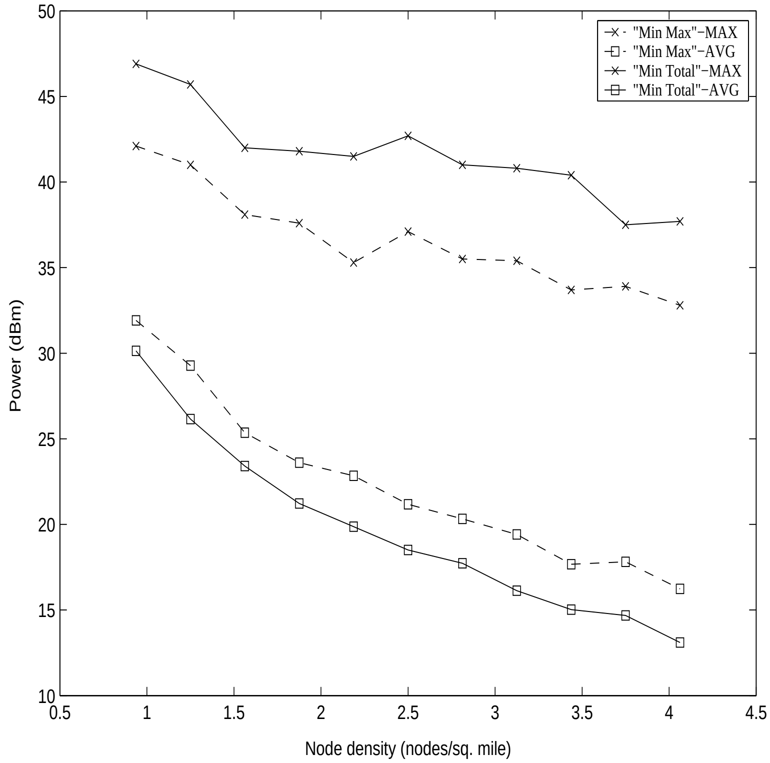
Uniform Node Density (continued)



Degree vs Node density

- Average node degree consistently smaller than [RR, 2000].

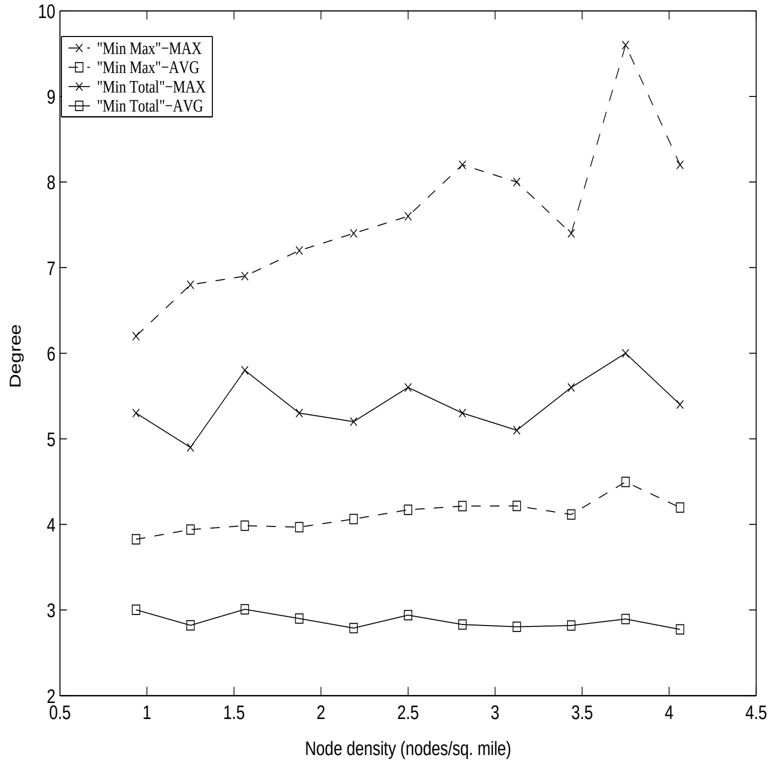
Skewed Node Density



Power vs Node density

- Average power less than that of [RR, 2000] by 6% to 14%.
- Maximum power is 12% to 20% larger.

Skewed Node Density (continued)



Degree vs Node density

- Average node degree consistently smaller.

Additional Recent Results

- Approximation algorithm for minimizing total power for inducing a network of small diameter.
- An $O(\log n)$ approximation algorithm for minimizing the total power for inducing a connected network under *asymmetric* power threshold values.