- Question 1: Soundness. Consider the following proposed inference rule.

  $\dfrac{p \rightarrow q}{q \rightarrow p}$ Indicate why this rule is or is not sound by identifying an associated implication and stating whether it is or is not a tautology.

Not sound. $(p \rightarrow q) \rightarrow (q \rightarrow p)$ *would be the required tautology, and it is not (let p be false and q be true)*

- Question 2: Consider the theorem: **If x^3 is irrational, then x is irrational.** This theorem has the form of an implication. For the 2 special cases in which something is assumed about x, there is a type of proof that establishes the theorem for that case immediately (or almost immediately). Circle that type of proof in each case.

  x = -1 DIRECT PROOF, VACUOUS PROOF, TRIVIAL PROOF
  ^^^^^^^^^^^^^^^^^^ (because $-1^3 = -1$ which is NOT irrational)

  $x = \sqrt{2}$. DIRECT PROOF, VACUOUS PROOF, TRIVIAL PROOF
  ^^^^^^^^^^^^^^^^ (x IS irrational)

- Question 3: The theorem below can be proved by induction. Show that the base case holds.

  $\sum_{i=0}^{n} 3 \cdot 5^n = 3 \cdot (5^{n+1} - 1)/4$ Basis: n=0. LHS= $3 \cdot 5^0 = 3$; RHS = $3 \cdot (5^{0+1} - 1)/4 = 3 \cdot (5 - 1)/4 = 3 \cdot 4/4 = 3$.

- Question 4: Consider this "theorem": **For n ≥ 1, the members of a set of n horses are all the same color.**
  Proof: By induction on n. Base case: n = 1. A set with one horse has only one color.
  Induction step: Assume the theorem for n = k. For n = k+1, let H be a set of k+1 horses.
  Remove 1 horse; in the resulting set of k horses, all have the same color by the Ind. Hyp. Replace that horse and remove another. This also results in a set of k horses, all of the same color. These two sets share k-1 horses and so both sets are of the same color. But their union is the original set of k+1 horses and thus all have one color. Q.E.D.

Indicate where the error is in this proof.

a) Base Case b) The induction step in general (e.g., going from 17 to 18 horses) c) The ind. step where n = 2.
^^^^^^^^^^^^^^^^^^^^^^^^^^

c) Because when n=2=k+1, k=1 and the two sets of horses share k-1 horses --- oops, they share 1-1=0 horses! So we can't conclude anything about the horses they don't share being the same color!