Variable elimination

\[ P(c) = \sum_a \left( \sum_b P(c \mid b) \sum_a P(b \mid a) P(a) \right) P(b) \]

Diagram: A -> B -> C

Summation: \( \sum_A \) and \( \sum_B \)
Variable Elimination with loops

P(A)  P(G)  P(S | A,G)
       \downarrow   \downarrow
P(A,G,S) \rightarrow \sum_{G} \rightarrow P(A,S) \rightarrow \sum_{E,S} \rightarrow P(A,E,S)

P(E | A)
P(C | E,S)
P(L | C)
P(C,L) \rightarrow \sum_{C} \rightarrow P(L)

Complexity is exponential in the size of the factors
Three Main Approaches To Approximate Inference

- MCMC
- Variational Methods
- Loopy belief propagation
Decision Making With BNets

Should I have my party inside or outside?

- **in**
  - **dry** Regret
  - **wet** Relieved
- **out**
  - **dry** Perfect!
  - **wet** Disaster
Minimize Risk

choose the action that maximizes expected utility
Influence Diagrams

Earthquake → Alarm → Newcast

Burglary → Call

Go Home? (Neighbors phoned Yes, No Phone Call No)

Miss Meeting → Big Sale

Goods Recovered → Utility

Expected Utility of this policy is 100
Putting a Price on Information

- What is it worth to get another piece of information?
- What is the increase in (maximized) expected utility if I make a decision with an additional piece of information?
- Additional information (if free) cannot make you worse off.
- There is no value-of-information if you will not change your decision.
Improvement With Extra Link

Expected Utility of this policy is 112.5
Dynamic Belief Networks

Markov property:

- past independent of future given current state;
- a conditional independence assumption;
- implied by fact that there are no arcs $t \rightarrow t+2$. 
State described via random variables.

Each variable depends only on few others.
HMM Algorithms Applied to DBNET

- HMMs are just very simple DBNs.
- Standard inference & learning algorithms for HMMs are instances of DBN algorithms
  - Forward-backward = polytree
  - Baum-Welch = EM
  - Viterbi = most probable explanation.
Next Lecture

• Major criticism of B Nets as models of human behavior
• Learning Bayes Nets
  – Parameter estimation
  – Learning structure