Practice Problems
Some Answers

1. For each of the following regular expressions over the alphabet \{a, b\}, exhibit a string, over \{a, b\}, that does not belong to the corresponding language:

   (a) \((baa^*)^* \cup a^*b^*\)

   ```
   % echo "aba" | egrep '^-((baa*)*|a*b*)$'
   
   %
   ```

   (b) \((a^*b(aa^*)^*)^* \cup a^*\)

   ```
   % echo "ba" | egrep '^-((a*|b(aa)*)*$'
   
   %
   ```

2. Show that the following context-free grammars are ambiguous. In every case, the start symbol is \(<S>\) and the terminal alphabet is \{a, b\}.

   (a) \(<S> ::= <S>a<S> \mid b\)

   \[\text{babab has 2 distinct derivation trees.}\]

   ![babab derivation trees]

   (b) \(<S> ::= a<S>S \mid a \mid aa\)

   (c) \(<S> ::= a<S>b \mid <S>b \mid <empty>\)

   (d) \(<S> ::= a<S><S>bb \mid <empty>\)

   \[\text{aabbbb has 2 distinct derivation trees.}\]
(e) \( \langle S \rangle ::= a\langle S\rangle b\langle S\rangle \mid b\langle S\rangle \mid a \)

\( a(ba)^4 \) has 2 distinct derivation trees.

(f) \( \langle S \rangle ::= a\langle S\rangle a b \mid a\langle S\rangle b \mid a a\langle S\rangle a \mid \langle \text{empty} \rangle \)

\( a^4abab \) has 2 distinct derivation trees.