1. State whether the following quantified formulae are true over the natural numbers $\mathbb{N} = \{1, 2, \cdots\}$:

   (i) $\forall x \exists y \exists z \left[ x + 1 = y^2 + z^2 \right]$

   (ii) $\forall x \forall y \exists z \left[ (x > y) \rightarrow (x^3 < y^3 + z^3) \right]$

   (iii) $\forall u \forall v \exists w \exists x \left[ ux < vw \right]$

   Give your reasons in each case. (No formal proof is needed.)

2. Exhibit a language $A$ over the alphabet $\{a, b\}$ such that $|A| = 4$ and $|A^2| = 13$.

   (Note that $A^2 = A \circ A$.)

3. Exhibit finite languages $A$ and $B$ such that $|A \circ B| < |B \circ A|$.