CSI 409 — Fall 2015: Homework #1

Due date: Sep 4

Answer all questions on your own. Turn in your answers at the beginning of class. Write your preferred e-mail address (e.g. zz6000@csc). If you are using more than one sheet of paper, make sure that you staple all the sheets together.

Remember that collaboration of any kind is not allowed.

1. State whether the following quantified formulae are true over the natural numbers \( \mathbb{N} = \{1, 2, \cdots\} \):
   
   (i) \( \forall x \exists y \forall z [xy \neq z^2] \)
   
   (ii) \( \exists x \forall y \exists z [(x = y^2) \lor (y = z^2)] \).
   
   (iii) \( \forall x \exists y \forall z \exists w [\lvert x - z \rvert \geq \lvert y - w \rvert] \)

   Give your reasons in each case. (No formal proof is needed.)

2. Use induction on \( n \) to prove that for all \( n \geq 2 \), \( 2^n + 3^n < 5^n \).

3. Let \( n \) be a natural number. Show that any set \( S \) of natural numbers of cardinality \( n \) (i.e., \( |S| = n \)) has a subset \( S' \) the sum of whose elements is a multiple of \( n \). (For instance, consider the set \( \{3, 11, 13, 18, 21\} \). Elements of the subset \( \{3, 11, 21\} \) add up to 35 which is a multiple of 5.)