CSI 409 — Fall 2017: Homework #1

Due date: Sep 15

Answer all questions on your own. Turn in your answers at the **beginning** of class. Write your preferred e-mail address (e.g. `zz6000@csc`). If you are using more than one sheet of paper, make sure that you *staple* all the sheets together.

Remember that collaboration of any kind is not allowed.

1. State whether the following quantified formulae are true over the natural numbers $\mathbb{N} = \{1, 2, \cdots\}$:

   (i) $\forall x\ \exists y\ \exists z\ \left[x+1 = y^2 + z^2\right]$

   (ii) $\forall x\ \forall y\ \exists z\ [(x > y) \rightarrow (x^3 < y^3 + z^3)]$

   (iii) $\forall u\ \forall v\ \exists w\ \exists x\ [ux < vw]$

   Give your reasons in each case. (No formal proof is needed.)

2. Exhibit a language A over the alphabet $\{a, b\}$ such that $|A| = 4$ and $|A^2| = 13$.

   (Note that $A^2 = A \circ A$.)

3. Exhibit finite languages A and B such that $|A \circ B| < |B \circ A|$.