CSI 409 — Fall 2017: Homework #1
Some answers and hints

1. State whether the following quantified formulae are true over the natural numbers $\mathbb{N} = \{1,2,\cdots\}$:

   (i) $\forall x \, \exists y \, \exists z \, \left[x+1 = y^2+z^2\right]$

   False: Take $x = 2$. Now 3 is not the sum of two squares.

   (ii) $\forall x \, \forall y \, \exists z \, [(x > y) \rightarrow (x^3 < y^3+z^3)]$

   True: Pick $z = x+1$.

   (iii) $\forall u \, \forall v \, \exists w \, \exists x \, [ux < vw]$

   True: Take $w = u+1$ and $x = 1$.

2. Exhibit a language A over the alphabet $\{a,b\}$ such that $|A| = 4$ and $|A^2| = 13$.

   (Note that $A^2 = A \circ A$.)

   $\{a,aa,aaaa,b\}$.

3. Exhibit finite languages A and B such that $|A \circ B| < |B \circ A|$.

   $A = \{a,aa\}, \; B = \{b,ab\}$.