1. Construct a deterministic finite automaton (DFA) that recognizes the following language:

\[ \{ w \in \{a,b\}^* \mid w \text{ starts with } b \text{ and contains } bb \text{ as a substring.} \} \]

The alphabet is \( \{a,b\} \).

**Note:** \( bb \) is in the language and so should be accepted by the DFA.

![DFA Diagram]

2. Consider the language 

\[ a^*b \cup b^* \]

(i.e., \( \{a\}^*\{b\} \cup \{b\}^* \)).

The alphabet is \( \{a,b\} \).

(a) Construct a deterministic finite automaton (DFA) recognizing this language.

Done in class.

(b) Show that any DFA that accepts this language has to contain a dead state.

**Hint:** Find a string \( w \) such that any string that has \( w \) as a prefix will not be in the language. It is not enough to exhibit one DFA for this language that has a dead state.

Done in class: any string that has \( ba \) as a prefix is not in the language.