1. Derive a regular expression for the complement of the language \((ab)^* \cup b\). The alphabet is \(\{a, b\}\).

The DFA for this language is:

![DFA Diagram]

The equations are:

\[
\begin{align*}
X_1 &= aX_2 \cup bX_4 & (1) \\
X_2 &= aX_5 \cup bX_3 \cup \epsilon & (2) \\
X_3 &= aX_2 \cup bX_5 & (3) \\
X_4 &= (a \cup b)X_5 & (4) \\
X_5 &= (a \cup b)X_5 \cup \epsilon & (5)
\end{align*}
\]

From the last two equations we get \(X_5 = (a \cup b)^*\) and thus \(X_4 = (a \cup b)^+\). Substituting for \(X_5\) in (3), we get \(X_3 = aX_2 \cup b(a \cup b)^*\). Now substituting for \(X_3\) in (2) gives us \(X_2 = a(a \cup b)^* \cup baX_2 \cup bb(a \cup b)^* \cup \epsilon\). Rearranging, this becomes

\[X_2 = baX_2 \cup (a \cup bb)(a \cup b)^* \cup \epsilon\]

and, by Arden’s Lemma, \(X_2 = (ba)^* \left[(a \cup bb)(a \cup b)^* \cup \epsilon\right]\). Thus finally

\[X_1 = a(ba)^* \left[(a \cup bb)(a \cup b)^* \cup \epsilon\right] \cup b(a \cup b)^+\].
2. Derive a regular expression for the following language:

\[ \{ w \mid |w| \text{ is odd and } w \text{ starts and ends with symbol } b \} \]

The alphabet is \{a, b\}. Note that the string b is in the above language.

The DFA for this language is:

![DFA Diagram]

The equations are

\[
X_1 = aX_2 \cup bX_3 \quad (6)
\]
\[
X_2 = (a \cup b)X_2 \quad (7)
\]
\[
X_3 = (a \cup b)X_4 \cup \varepsilon \quad (8)
\]
\[
X_4 = aX_5 \cup bX_3 \quad (9)
\]
\[
X_5 = (a \cup b)X_4 \quad (10)
\]

We get \(X_2 = \emptyset\) from (7). Thus \(X_1 = bX_3\). Replacing \(X_5\) in (9) by \((a \cup b)X_4\) and then applying Arden’s Lemma, we get \(X_4 = (aa \cup ab)^*bX_3\). Therefore (from (8)) \(X_3 = (a \cup b)(aa \cup ab)^*bX_3 \cup \varepsilon\). Thus \(X_3 = ((a \cup b)(aa \cup ab)^*b)^*\) and finally

\[
X_1 = b \left( (a \cup b)(aa \cup ab)^*b \right)^* \quad (11)
\]

Alternatively, we can solve for \(X_4\) first:

\[
X_4 = a(a \cup b)X_4 \cup b(a \cup b)X_4 \cup b \quad (12)
\]

from (8), (9) and (10). Thus \(X_4 = (aa \cup ab \cup ba \cup bb)^*b\). Now

\[
X_1 = bX_3 = b(a \cup b)X_4 \cup b = b(a \cup b) \left( aa \cup ab \cup ba \cup bb \right)^*b \cup b \quad (13)
\]

We get \(X_2 = \emptyset\) from (7). Thus \(X_1 = bX_3\). Replacing \(X_5\) in (9) by \((a \cup b)X_4\) and then applying Arden’s Lemma, we get \(X_4 = (aa \cup ab)^*bX_3\). Therefore (from (8)) \(X_3 = (a \cup b)(aa \cup ab)^*bX_3 \cup \varepsilon\). Thus

\[
X_3 = ((a \cup b)(aa \cup ab)^*b)^* \quad (11)
\]

Alternatively, we can solve for \(X_4\) first:

\[
X_4 = a(a \cup b)X_4 \cup b(a \cup b)X_4 \cup b \quad (12)
\]

from (8), (9) and (10). Thus \(X_4 = (aa \cup ab \cup ba \cup bb)^*b\). Now

\[
X_1 = bX_3 = b(a \cup b)X_4 \cup b = b(a \cup b) \left( aa \cup ab \cup ba \cup bb \right)^*b \cup b \quad (13)
\]