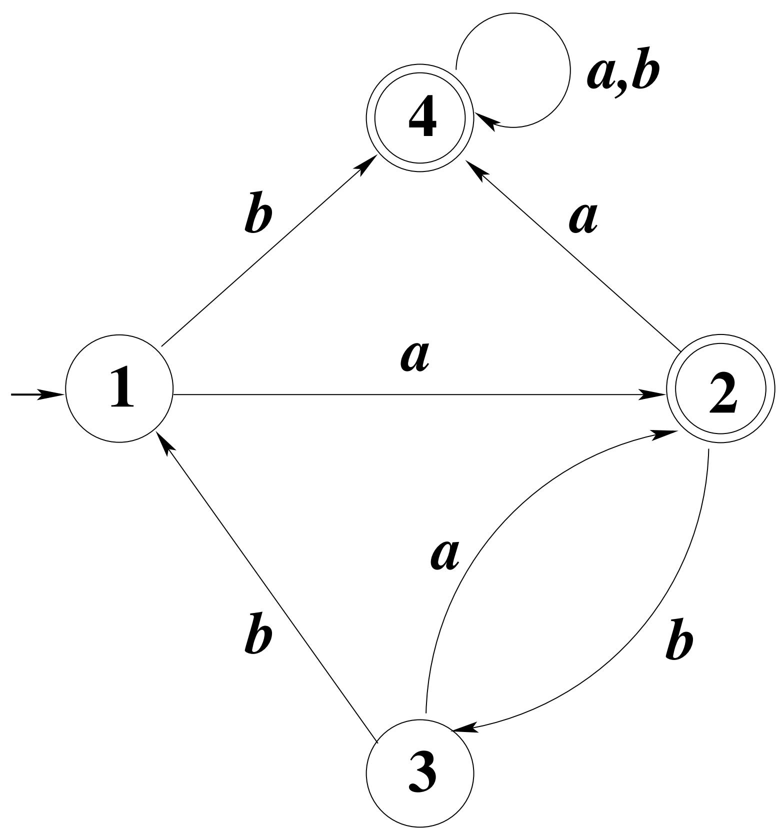
CSI 409 — Fall 2017: Homework #3
Some answers and hints

1. Derive a regular expression for the *complement* of the language $(ab \cup abb)^*$. The alphabet is $\{a,b\}$.



The equations are:

$$
\begin{aligned}
X_1 &= aX_2 \cup bX_4 \\
X_2 &= aX_4 \cup bX_3 \cup \varepsilon \\
X_3 &= aX_2 \cup bX_1 \\
X_4 &= aX_4 \cup bX_4 \cup \varepsilon
\end{aligned}
$$

By Arden's Lemma, $X_4 = (a \cup b)^*$. The second equation can now be rewritten as

$$X_2 = a(a \cup b)^* \cup baX_2 \cup bbX_1 \cup \varepsilon$$

Applying Arden's Lemma to this equation, we get

$$X_2 = (ba)^*\big(a(a \cup b)^* \cup bbX_1 \cup \varepsilon\big) = (ba)^*a(a \cup b)^* \cup (ba)^*bbX_1 \cup (ba)^*$$

Thus the first equation becomes

$$X_1 = a(ba)^*a(a \cup b)^* \cup a(ba)^*bbX_1 \cup a(ba)^* \cup b(a \cup b)^*$$

Rearranging terms,

$$X_1 = a(ba)^*bbX_1 \cup \Big(a(ba)^*a(a \cup b)^* \cup a(ba)^* \cup b(a \cup b)^*\Big)$$

Thus the answer is, by Arden's Lemma,

$$\big(a(ba)^*bb\big)^*\Big(a(ba)^*a(a \cup b)^* \cup a(ba)^* \cup b(a \cup b)^*\Big)$$

2. Prove that the following language is not regular:

$$\Big\{a^m ba^n \;\Big|\; 2m > n \geq 0\Big\}$$

Take $w = a^p ba^{2p-1}$ and remove the pump.