CSI 409 — Fall 2017: Homework #4
Some answers and hints

1. Design a context-free grammar for the language

$$L = \left\{ a^k b^m \;\middle|\; (0 \le k < m) \vee (k > 2m \ge 0) \right\}$$

The alphabet is $\{a, b\}$.

Hint: The strings a, b, abb, $aaab$ and $bbbb$ belong to L, whereas ε, $aabb$ and $aaabb$ do not.

$$\begin{array}{rcl} S & \to & X \mid Y \\ X & \to & Xb \mid aXb \mid b \\ Y & \to & aaYb \mid aY \mid a \end{array}$$

2. Design a context-free grammar for the language

$$L = \left\{ a^k b^m \;\middle|\; k+m \text{ is an even number} \right\}$$

The alphabet is $\{a, b\}$.

Hint: The strings ε, aa, bb, $aabb$, $aaab$ and $bbbb$ belong to L, whereas a, b, $abab$ and $aaabb$ do not.

$$\begin{array}{rcl} S & \to & X \mid aXb \\ X & \to & aaX \mid Xbb \mid \varepsilon \end{array}$$