

## CSI 409 — Fall 2017: Homework #6

### Some answers and hints

1. Prove that the language

$$\{a^m b^m c^n \mid n > 2m \geq 0\}$$

is not context-free.

Take  $z = a^p b^p c^{2p+1}$  where  $p$  is the pumping length. By the pumping lemma, there are  $u, v, w, x, y$  such that  $z = a^p b^p c^{2p+1} = uvwxy$ , where  $v$  and  $x$  are respectively the left and right pumps. Additionally,  $1 \leq |vwx| \leq p$ .

Now if  $vwx$  is a substring of  $a^p b^p$  then pump up. If it's a substring of  $b^p c^{2p+1}$  then remove the pumps.

2. Prove that the language

$$\{a^i b^j \mid (i^2 > j \geq 0)\}$$

is not context-free.

Take  $z = a^{p^2+1} b^{p^2+2p}$  where  $p$  is the pumping length. By the pumping lemma, there are  $u, v, w, x, y$  such that  $a^{p^2+1} b^{p^2+2p} = uvwxy$ , where  $v$  and  $x$  are respectively the left and right pumps. Additionally,  $1 \leq |vx| \leq p$ .

Now if  $vwx$  is a substring of  $b^{p^2+2p}$ , then pump up. This increases the number of  $b$ 's, but the number of  $a$ 's stays the same.

If  $vwx$  contains an  $a$ , then remove the pumps. Let  $z' = uwy$ . Now  $\#_a(z') \leq p$  and  $\#_b(z') = p^2 + 2p - k$  for some  $0 \leq k \leq p$ . The value of  $p^2$  will always be less than  $p^2 + 2p - k$  since  $k \leq p$ .