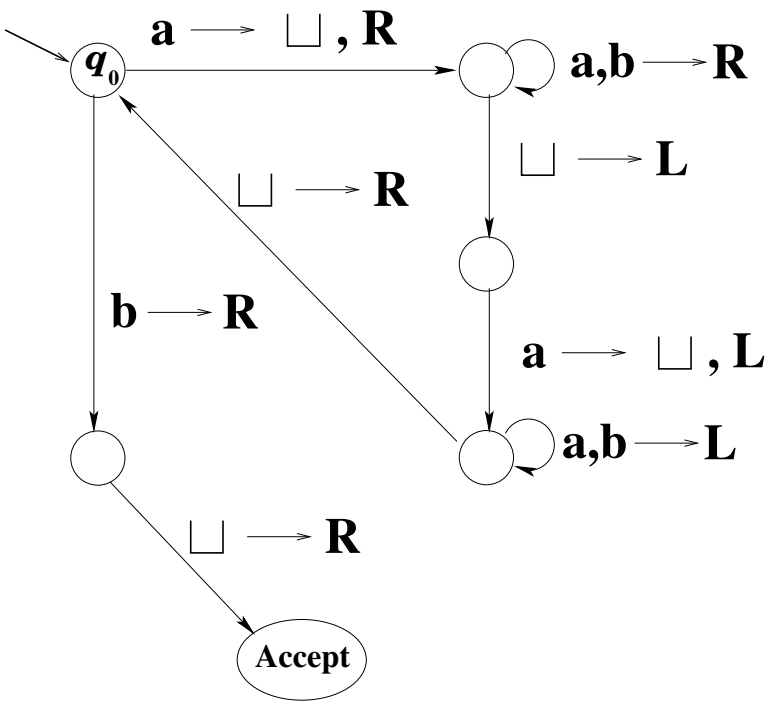
CSI 409 — Fall 2017: Homework #7
Some answers and hints

1. Consider the following Turing Machine. Its input alphabet is $\{a,b\}$. Transitions to the REJECT state are not shown. ("If stuck, reject.")



(a) Exhibit a non-empty string over $\{a,b\}$ that this TM accepts.

aba

(b) Exhibit a non-empty string over $\{a,b\}$ that this TM does not accept.

ab

(c) What language does this TM accept? (You should characterize it in terms of properties of the accepted strings.)

$$\left\{ a^m b a^m \;\middle|\; m \geq 0 \right\}$$

see next page

2. Exhibit a derivation of the string $a^2b^4c^2$ in the following grammar:

$$\begin{aligned} S &\rightarrow SABBC \mid X \\ CB &\rightarrow BC \\ CA &\rightarrow AC \\ BA &\rightarrow AB \\ XA &\rightarrow aX \mid aY \\ YB &\rightarrow bY \mid bZ \\ ZC &\rightarrow cZ \mid c \end{aligned}$$

What language does this grammar generate?

$S \Rightarrow SABBC \Rightarrow SABBCABBC \Rightarrow XABBCABBC \Rightarrow XABBACBBC \Rightarrow XABABCBBC$
$\Rightarrow XABABBCBC \Rightarrow XABABBBCC \Rightarrow XAABBBBCC \Rightarrow aXABBBBCC \Rightarrow aaYBBBBCC$
$\Rightarrow aabYBBBCC \Rightarrow aabbYBBCC \Rightarrow aabbbYBCC \Rightarrow aabbbbZCC \Rightarrow aabbbbcZC$
$\Rightarrow aabbbbcc$

The language is $\left\{ a^m b^{2m} c^m \;\middle|\; m > 0 \right\}$.