CSI 409 — Fall 2017: Midterm Exam #I
Some answers and hints

1. Exhibit languages L_1 and L_2 such that $L_1 \not\subseteq L_2$, $L_2 \not\subseteq L_1$ *and* $L_1^* = L_2^*$.

   (In other words, L_1 and L_2 are incomparable by the subset relation $\subseteq$, but L_1^* and L_2^* are the same.)

   Take $L_1 = \{a, aa\}$ and $L_2 = \{a, aaa\}$. Now $L_1^* = L_2^* = \{a\}^*$.

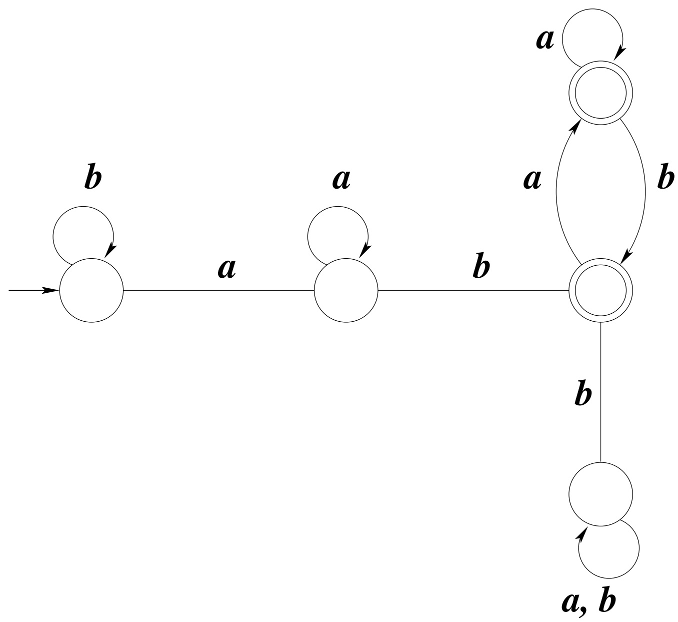
2. Show that the regular expressions $(ab)^*a^*$ and $a^*(ab)^*$ are *not equivalent.*

   (That is, explain why the languages $\{ab\}^*\{a\}^*$ and $\{a\}^*\{ab\}^*$ are not the same.)

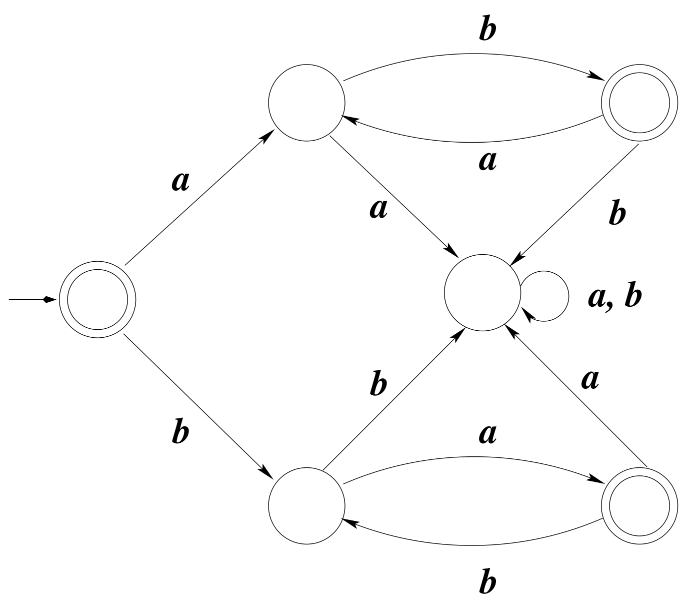
   $\{ab\}^*\{a\}^*$ does not contain the string aab. First of all, aab does not belong to either $\{ab\}^*$ or $\{a\}^*$. Since every non-empty string in $\{a\}^*$ ends with a, we reach an impossibility.

3. Construct deterministic finite automata (DFA) that recognize the following languages. The alphabet is $\{a,b\}$:

(a) (17 points) $\{ w \mid ab$ is a substring of w, but abb is not $\}$.

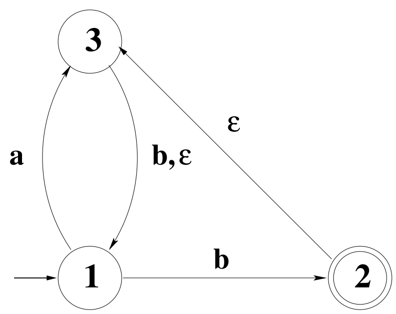


(b) (17 points) $(ab)^* \cup (ba)^*$.



4. Consider the following NFA. The set of states, Q, is $\{1,2,3\}$. The initial state is 1 and the accepting state is 2. The alphabet is $\{a,b\}$.

	a	b	ε
1	$\{3\}$	$\{2\}$	$\{\}$
2	$\{\}$	$\{\}$	$\{3\}$
3	$\{\}$	$\{1\}$	$\{1\}$



Convert this NFA to a DFA. Show work clearly.