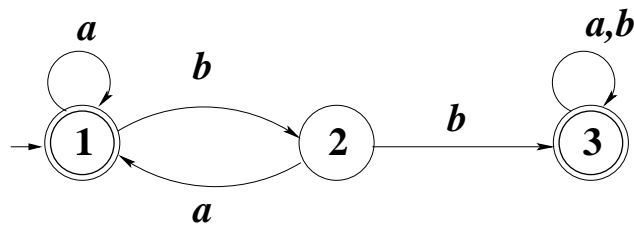


CSI 409: Sample problems on context-free languages

1. Construct a context-free grammar for the *complement* of the language $(a \cup ba)^*b$. (The alphabet is $\{a, b\}$.)

The DFA is



Thus the (right-linear) grammar is

$$\begin{aligned} S &\rightarrow aS \mid bA_2 \mid \varepsilon \\ A_2 &\rightarrow aS \mid bA_3 \\ A_3 &\rightarrow aA_3 \mid bA_3 \mid \varepsilon \end{aligned}$$

2. Exhibit a context-free grammar for the language

$$L = \{a^k b^m \mid 0 \leq k \leq 2m\}$$

The alphabet is $\{a, b\}$.

$$\begin{aligned} S &\rightarrow AASb \mid \varepsilon \\ A &\rightarrow a \mid \varepsilon \end{aligned}$$

Note that k and m can be 0, thus ε is in the language.

3. Exhibit a context-free grammar for the language

$$L = \{a^k b^m \mid 2k > m \geq 0\}$$

The alphabet is $\{a, b\}$.

$$\begin{aligned} S &\rightarrow aSBB \mid ab \mid a \\ B &\rightarrow b \mid \varepsilon \end{aligned}$$

4. Show that the language

$$\{wcw^R \mid w \in a^+b^+\}$$

is context-free. The alphabet is $\{a, b, c\}$.

$$\begin{aligned} S &\rightarrow aSa \mid aXa \\ X &\rightarrow bXb \mid bcb \end{aligned}$$