

## CSI 409: Some sample problems on regular languages and DFAs

1. Disprove the following:  $A \circ (B \cap C) = (A \circ B) \cap (A \circ C)$  for all languages  $A, B, C$ .

(Exhibit languages  $A, B, C$  such that  $A \circ (B \cap C) \neq (A \circ B) \cap (A \circ C)$ .)

2. Disprove the following: for all languages  $A, B$

$$A \subsetneq B \rightarrow A^* \subsetneq B^*$$

(In other words, exhibit languages  $A$  and  $B$  such that  $A$  is a proper subset of  $B$ , but  $A^*$  is not a *proper* subset of  $B^*$ .)

3. Exhibit a string over the alphabet  $\{a, b\}$  that does **not** belong to  $(a \cup ba)^* b^* a^*$ . Justify your answer. (No formal proof is needed.)
4. Construct deterministic finite automata (DFAs) recognizing the following languages over the alphabet  $\{a, b\}$ :

(a)  $\{a^i b^j \mid i \geq 0, j \geq 0, i + j \text{ is an even number}\}$

(0 is an even number, so  $\epsilon$  is in this language.)

(b) The set of all strings that begin with  $a$  but do not contain  $aab$  as a substring.

(c)  $(ab)^* \cup a$

(d)  $b(aa)^*$  (i.e.,  $\{b\} \circ \{aa\}^*$ )

5. Consider the language  $a^*b \cup b^*$  over the alphabet  $\{a, b\}$ . Show that any DFA that accepts this language *has* to contain a **dead state**.

**Hint:** Find a string  $w$  such that *any* string that has  $w$  as a prefix will not be in the language. It is not enough to exhibit one DFA for this language that has a dead state.