# CSI 409: DFA design problems <br> Some answers and hints 

1. Disprove the following: $A \circ(B \cap C)=(A \circ B) \cap(A \circ C)$ for all languages $A, B, C$.

Take $A=\{\epsilon, a\}, B=\{\epsilon\}, C=\{a\}$.
2. Disprove the following: for all languages $A, B$

$$
A \subsetneq B \rightarrow A^{*} \subsetneq B^{*}
$$

Take $A=\{a\}$ and $B=\{\epsilon, a\}$.
3. Construct deterministic finite automata (DFAs) recognizing the following languages over the alphabet $\{a, b\}$ :
(a) $\left\{a^{i} b^{j} \mid i \geq 0, j \geq 0, i+j\right.$ is an even number $\}$

(b) The set of all strings that begin with $a$ but do not contain $a a b$ as a substring.

(c) $(a b)^{*} \cup a$

(d) $b(a a)^{*}$

4. Consider the language $a^{*} b \cup b^{*}$ over the alphabet $\{a, b\}$. Show that any DFA that accepts this language has to contain a dead state.

No string that has $b a$ as a prefix is in the language.

