1. Construct deterministic finite automata (DFAs) recognizing the following languages over the alphabet \{a, b\}:

(a) \{w \mid aa \text{ is a substring of } w, \text{ but } ab \text{ is not}\}

(For instance, baa is in this language, but aab is not.)
(b) \( \{a^i b^j \mid i \geq 0, j \geq 0, i + j \text{ is an even number} \} \)
(c) The set of all strings that begin with $a$ but do not contain $aab$ as a substring.

(d) $\{ab\}^* \cup \{a\}$
2. Disprove the following: for all languages $A, B$

$$A \subseteq B \implies A^* \subseteq B^*$$

Take $A = \{a\}$ and $B = \{\epsilon, a\}$.

3. Disprove the following: $A \circ (B \cap C) = (A \circ B) \cap (A \circ C)$ for all languages $A, B, C$.

Take $A = \{\epsilon, a\}$, $B = \{\epsilon\}$, $C = \{a\}$.