Exercises on phrase-structured grammars

1. Consider the following grammar:

   \[
   \begin{align*}
   S & \rightarrow aSA \mid bSb \mid \\
   bA & \rightarrow Ab \\
   \#A & \rightarrow \#a \\
   aA & \rightarrow aa
   \end{align*}
   \]

   The alphabet is \{a, b, \#\} and the start symbol is \(S\).

   Show a derivation of the string \(abba\#aabb\) using this grammar. What language does this grammar generate?

2. Consider the following phrase-structure grammar:

   \[
   \begin{align*}
   S & \rightarrow Abc \\
   A & \rightarrow aAbC \mid AbC \mid \epsilon \\
   Cb & \rightarrow bc \\
   Cc & \rightarrow cc
   \end{align*}
   \]

   The alphabet is \{a, b, c\} and the start symbol is \(S\).

   Show a derivation of the string \(abbbccc\) using this grammar. What language does this grammar generate?

3. Consider the following phrase-structure grammar:

   \[
   \begin{align*}
   S & \rightarrow AASB \mid \epsilon \\
   AB & \rightarrow BA \\
   A & \rightarrow a \\
   B & \rightarrow b
   \end{align*}
   \]

   The alphabet is \{a, b\} and the start symbol is \(S\).
Show a derivation of the string $ababaa$ using this grammar. What language does this grammar generate?

4. Consider the following grammar:

$$
S \rightarrow aSBc \mid SBc \mid Sc \mid abc \\
cB \rightarrow Bc \\
bB \rightarrow bb
$$

The alphabet is \{a, b\} and the start symbol is $S$.

Show a derivation of the string $abbc$ using this grammar. What language does this grammar generate?