Consider Example 1.68 (page 76 of the textbook). We get the equations

$$X_1 = aX_2 \cup bX_3 \tag{1}$$
$$X_2 = aX_1 \cup bX_2 \cup \epsilon \tag{2}$$
$$X_3 = aX_2 \cup bX_1 \cup \epsilon \tag{3}$$

Eliminating X_1 from equations (2) and (3), we get

$$X_2 = (aa \cup b)X_2 \cup abX_3 \cup \epsilon \tag{4}$$
$$X_3 = (a \cup ba)X_2 \cup bbX_3 \cup \epsilon \tag{5}$$

Applying Arden's Lemma to (4):

$$X_2 = (aa \cup b)^*(abX_3 \cup \epsilon) \tag{6}$$
$$= (aa \cup b)^*abX_3 \cup (aa \cup b)^* \tag{7}$$

Thus

$$X_3 = (a \cup ba)X_2 \cup bbX_3 \cup \epsilon$$
$$= (a \cup ba)\underbrace{((aa \cup b)^*abX_3 \cup (aa \cup b)^*)} \cup bbX_3 \cup \epsilon \tag{8}$$
$$= \underline{(a \cup ba)(aa \cup b)^*ab}X_3 \cup (a \cup ba)(aa \cup b)^* \cup \underline{bb}X_3 \cup \epsilon \tag{9}$$
$$= (bb \cup (a \cup ba)(aa \cup b)^*ab)X_3 \cup (a \cup ba)(aa \cup b)^* \cup \epsilon \tag{10}$$

Applying Arden's Lemma to (10), we get

$$X_3 = (bb \cup (a \cup ba)(aa \cup b)^*ab)^*((a \cup ba)(aa \cup b)^* \cup \epsilon) \tag{11}$$

Thus we have solved for X_3. However, our goal is X_1. But note that X_1 can be expressed entirely in terms of X_3 using (1) and (7):

$$X_1 = aX_2 \cup bX_3$$
$$= a\underbrace{((aa \cup b)^*abX_3 \cup (aa \cup b)^*)} \cup bX_3 \tag{12}$$
$$= (a(aa \cup b)^*ab \cup b)X_3 \cup a(aa \cup b)^* \tag{13}$$

Substituting for X_3 in (13), we get the horrendous

$$X_1 = (a(aa \cup b)^*ab \cup b)((bb \cup (a \cup ba)(aa \cup b)^*ab)^*((a \cup ba)(aa \cup b)^* \cup \epsilon)) \cup a(aa \cup b)^*$$