Solution of Problem 1(a):

Let -X be the additive inverse of X. That is \(-X \oplus X = 0\). Then:

\[ C = (P \oplus K_r) \otimes K_l \]

\[ C = ((P \oplus K_r) + K_l) \text{ mod } 2^{32} \]

\[ C - K_l = (P \oplus K_r) \text{ mod } 2^{32} \quad \text{// By applying additive inverse property} \]

\[ (C - K_l) \text{ mod } 2^{32} = (P \oplus K_r) \]

\[ (C - K_l) \oplus K_r \text{ mod } 2^{32} = (P \oplus K_r) \oplus K_r \quad \text{// By XORing } K_r \text{ on both side} \]

\[ (C \otimes (-K_l)) \oplus K_r = P \quad \text{// Since } K_r \oplus K_r = 0 \]

Solution of Problem 1(b):

First, calculate \(-C'\). Then \(-C' = (P' \oplus K_l) \otimes (-K_l)\). We then have:

\[ C \otimes -C' = (P \oplus K_l) \otimes (P' \oplus K_l) \]

However, the operations \(\otimes\) and \(\oplus\) are not associative or distributive with one another, so it is not possible to solve this equation for \(K_l\).

Solution of Problem 2:

Say your name is JOHN MURRAY

\[ X = J + O + H + N \text{ mod } 26 = 9 + 14 + 7 + 13 \text{ mod } 26 = 17 \]

\[ Y = M + U + R + R + A + Y = 12 + 20 + 16 + 16 + 0 + 24 \text{ mod } 26 = 10 \]

Most frequent letter is E i.e. 4
Second most frequent letter is T. i.e. 19

So we have two equations:

\[ 17 = 4 \times a + b \text{ mod } 26 \quad (1) \]

\[ 10 = 19 \times a + b \text{ mod } 26 \quad (2) \]

\[ (1) - (2) \Rightarrow 7 = -15 \times a \text{ mod } 26 \]

\[ \Rightarrow 7 + 15 \times a \text{ mod } 26 = 0 \]

\[ \Rightarrow a = 3 \]
Putting \( a = 3 \) in (1)

\[
\Rightarrow 17 = 4 \times 3 + b \mod 26 \\
\Rightarrow 17 - 12 = b \mod 26 \\
\Rightarrow 5 = b \mod 26 \\
\Rightarrow b = 5
\]

So, with the given information, the cipher is breakable. We have found the key \( a = 3 \) and \( b = 5 \).