Note: Much of this assignment will review some mathematical concepts and skills that are needed in computer graphics. Its difficulty is typical of the math to be used and taught in this course. Feel free to use any reference you like to assist you in doing these problems, except do not copy answers from other people. If you don’t already know this material, look for help in Schaum’s Linear Algebra Outline or other math textbooks. Make sure to show all work.

1. (Calculating frame buffer storage size.) Exercise 2-4 in Hearn and Baker.

2. (Calculating speeds and access times.) Exercise 2-8 in Hearn and Baker.

3. (Read about 2 technologies and think critically about them.) Exercise 2-14 in Hearn and Baker. (200 words or less)

4. (On a library architecture.) Exercise 2-14 in Hearn and Baker. (200 words or less)

5. (On a system architecture...IMPORTANT!) Exercise 2-22 in Hearn and Baker. (200 words or less)

6. Demonstrate Bresenham’s algorithm to rasterize the line segment from screen coordinates (2, 3) to (12, 7). Show all the calculations. Draw the theoretical line and its rasterization on the grid below.

7. Plot or sketch on the grid below, or your own graph paper. Put the origin at the center. Scale so that grid lines are 1 unit apart. Use pencil so you can make corrections neatly.

(a) The points: (5, 3), (15, 4)

(b) The lines: $y = (3/5)x + 6, \ y = (-2)x - 3, \ y/5 + x/11 = 1.$
(c) The curve: \( y = (x/3)^2 - 4 \).

(d) Sketch the path traced out by the point \((x(t), y(t))\) as the parameter \(t\) varies from 
-1 to +2, where \(x(t) = 4t^2 - 1\) and \(y(t) = 2t\).

8. Write equations for each of the lines:

(a) Whose slope equals \(m\) and whose \(y\) intercept equals \(b\).

(b) Whose slope equals \(m\) and whose \(x\) intercept equals \(a\).

(c) Whose slope equals \(m\) and which passes through point \((x_0, y_0)\).

(d) That passes through the two points \((x_0, y_0)\) and \((x_1, y_1)\).

(e) Whose \(x\) intercept is \(a\) and whose \(y\) intercept is \(b\).

9. In these equations, \(xv_1, xv_2, xv_1, xw_1, xw_2, xw_1, yv_1, yv_2, yv_1, yw_1, yw_2, yw_1\) are con-
stants and \(xv, xw, yv, yw\) are variables.

\[
\frac{xv - xv_1}{xv_2 - xv_1} = \frac{xw - xw_1}{xw_2 - xw_1}
\]
\[
\frac{yv_2 - yv}{yv_2 - yv_1} = \frac{yw - yw_1}{yw_2 - yw_1}
\]

(a) Solve for \(xv\) and \(yv\) in terms of \(xw\) and \(yw\), and constants.

(b) Solve for \(xw\) and \(yw\) in terms of \(xv\) and \(yv\), and constants.

10. What is the length of the line segment defined by points \((138, 539)\) and \((134, 544)\)?
11. Multiply the following pairs of 2x2 matrices: (See the Linear Algebra Outline if necessary.)

\[
\begin{pmatrix}
1 & 2 \\
3 & 4
\end{pmatrix}
\begin{pmatrix}
4 & 3 \\
1 & 2
\end{pmatrix}
\]

\[
\begin{pmatrix}
4 & 3 \\
1 & 2
\end{pmatrix}
\begin{pmatrix}
1 & 2 \\
3 & 4
\end{pmatrix}
\]

12. A house floats in space as shown in the diagram. It is to be described using 3 dimensional coordinates. The base of the house lies 1 meter above the x-y plane. The front lowest left corner denoted by E lies over position (2,4) in the x-y plane. Therefore, the coordinates of corner E are \( x = 2, \ y = 4 \) and \( z = 1 \). This is denoted by (2, 4, 1). Write the 3-d coordinates of the four indicated corners A, B, C and D. The peak of the roof is centered between the front and back planes of the house. This peak lies 2 meters above the base of the roof.

13. Solve each of the following 2 systems of equations:

\[
\begin{align*}
4x + 3y &= 1 \\
1x + 2y &= 0
\end{align*}
\]

\[
\begin{align*}
4x + 3y &= 0 \\
1x + 2y &= 1
\end{align*}
\]

14. Suppose variables \( x', y' \) relate to \( x, y \) by the equations below:

\[
\begin{align*}
x' &= 3x + 4y + 1 \\
y' &= -4x + 3y + 2
\end{align*}
\]

Suppose \( x'', y'' \) relate to \( x', y' \) by the equations:

\[
\begin{align*}
x'' &= ax' + by' \\
y'' &= cx' + dy'
\end{align*}
\]

Substitute \( x', y' \) given by (1) into (2) and simplify so the result is expressed in the following form:

\[
x'' = Px + Qy + R
\]
\[ y'' = Sx + Ty + U \]

In other words, calculate formulas for \( P, Q, R, S, T \) and \( U \) in terms of numbers and variables \( a, b, c \) and \( d \).

15. For parts (a), (b) and (c), describe, using inequalities such as \( t \geq A, t \leq B, t < C \), etc. the values of \( t \) for which the function \( x(t) = 3t - 6 \) takes on values that are:

(a) Less than or equal to 0. \( x(t) \leq 0 \).

(b) Greater than or equal to 1. \( x(t) \geq 1 \).

(c) Between 0 and 1, exclusively \( 0 < x(t) < 1 \).