Univ. at Albany, SUNY CSI 422/502 Computer Graphics Fall 2005 Professor Chaiken
Homework 4 Due 10/21/05

The objective of this problem is to visualize the construction of pages 268-270 in HB.

The axis of rotation is the direction from $P_1 = (0, 0, 0)$ to $P_2 = (12, 3, 4)$. The construction will calculate a matrix whose transformation will rotate space so that P_2 is transformed to a point on the z-axis.

Read sections 7-5 and 7-6 in HB. Plot on on graph paper the three orthogonal projections of P_1, P_2 and the line between them:

1. (1) Onto the xy-plane looking from the positive z-axis toward the origin.
2. (2) Onto the xz-plane looking from the positive y-axis toward the origin.
3. (3) Onto the yz-plane looking from the positive x-axis toward the origin.

(4) Use the Pythagorian Theorem ($H^2 = A^2 + B^2$ for the lengths of the hypotenuse H and the sides A and B of a right triangle) on yz projection plot to calculate the length of the projection of P_1P_2 on the yz plane.

(5) Mark on your plot on the yz plane the angle between the projection of line P_1P_2 and the z-axis. (6) Write as a fraction the ratio of lengths that is the sine of this angle. (7) Write as a fraction the ratio of lengths that is the cosine of this angle.

(8) Form the rotation matrix that rotates by the above angle around the x axis.

(9) Verify that this matrix rotates the projection of P_1P_2 which you drew in the yz-plane into the z axis. (10) Verify that this rotation did not change the length of the projection in the yz-plane.

(11) What does this rotation do to the point P_2? (12) Calculate the matrix times the coordinates of P_2. We will use P_2' to denote the result of transforming P_2 with this rotation.

(13) Plot in the xz-plane the line P_1P_2', which is P_1P_2 after the above rotation is applied to it. (14) Verify that your plot of this line shows its **true length**. It should show the true length because the rotation will place the line so it is in the xz-plane.

(15) Now identify the angle in the xz-plane so rotating around the y axis by that angle will rotate P_1P_2' into the z axis. (16) As before, identify the ratios which are the sine and cosine of the angle. (17) As before, form the matrix that represents the rotation. Verify that when the P_1P_2' is transformed by this matrix, the result is a line segment that lies in the z-axis.

(18) Combine (by multiplying) the two matrices into one matrix that rotates line P_1P_2 into the z-axis.

(19) Verify your answer is correct by using your product matrix to transform $P_2 = (12, 3, 4)$ into point $(0, 0, 13)$ (which of course lies on the z-axis).