1 Problem 1

The objective of this problem is to experience the two alternate ways to understand a modeling transformation: The first is the transformation transforms one graphic object into another (by transforming vertices) and the second is the transformation transforms the coordinate systems used to describe or define fixed objects.

Let mF be the model of a displaced “F”: The 5 points A(7,0), B(7,2), C(8,2), D(7,4), E(10,4); and 3 lines AD, BC, DE.

1. Plot mf on graph paper.

We want to draw the instance of mF obtained by rotating the points and lines by 30° (counterclockwise) around the point A(7,0). Of course, this transformation is the composition

$$T_dR(30^\circ)T_{-d}$$

where \(d\) is the displacement \((7,0,0)\).

2. What are \(T_d\), \(R(30^\circ)\), and \(T_{-d}\)? Write their matrices.

Way 1: Transform objects

3. On a blank area of graph paper, plot the following objects: (0) mf. (1) \(T_{-d}(mF)\). (2) \(R(30^\circ)(T_{-d}(mF))\). (3) \(T_d(R(30^\circ)(T_{-d}(mF)))\).

Label the 4 F’s by 0-3 and verify that the 4th is the drawing we want. (Write on your paper whether or not it is verified!)

Way 2: Transform Coordinate Systems

4. On a blank area of graph paper:

- Draw coordinate axes and label them with (0).
- Draw the axes of the new coordinate system defined by transformation \(T_d\). Label these axes with (1).
- Repeat for the axes defined by transformation \(T_dR(30^\circ)\). Label these axes with (2).
- Repeat for the axes defined by transformation \(T_dR(30^\circ)T_{-d}\). Label these axes with (3).
- Carefully locate the point whose coordinates are A(7,0) interpreted under the 3rd system, whose axes are labelled by (3). Locate and label B, C, D, and E as well and draw the 3 lines of mF.

5. Verify (and write your conclusion) that your drawing, interpreted or viewed from the original coordinate system, are the same for both Way 1 and Way 2.
2 Problem 3

A better version of a model F would have the base A be the origin of the modelling coordinate system.

1. What are the model coordinates of points A-E in the better version?
2. What are the two transformations (a rotation and a translation) which when applied in sequence to the better model will produce the rotated F we wanted?
3. Draw the model and the two successive transformations of it to illustrate Way 1 as before.
4. Draw the axes of the original and the two transformed coordinate systems, and the model using the last system to illustrate Way 2 as before.

3 Problem 3

It was taught that the square of the length of the straight line from (0, 0, 0) to (A, B, C) is $A^2 + B^2 + C^2$. This is the 3-d variant of the Pythagorean Theorem.

Suppose you didn’t trust this statement but you accepted the original Pythagorean Theorem of plane geometry: If $H$ is the length of the hypotenuse of a right triangle and $S$ and $T$ are the lengths of the other two sides, then $H^2 = S^2 + T^2$.

Prove the 3-d variant by finding two right triangles some of whose sides have lengths $A$, $B$ and $C$. You will have to figure out what triangles to use. To make it clear what points in space you are taking about, write 3-d coordinates of those points. You will then have to substitute the sum of two squares for one square in a proved expression for the square of the length of the (0, 0, 0) to (A, B, C) line.

Hint: One of the right triangles will be in one of the coordinate planes (you can choose any of the 3 planes for it). The other triangle will be perpendicular to that plane.

To figure it out, or express your explanation, it might help draw a rough perspective sketch, and/or to draw orthogonal projections.