NON-BLIND IMAGE RESTORATION WITH SYMMETRIC GENERALIZED PARETO PRIORS

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Introduction

- Non-Blind Image Restoration

$$\text{Degraded Image} \rightarrow \boldsymbol{y} = \boldsymbol{x} \otimes \boldsymbol{k} + \boldsymbol{n} \leftarrow \text{Gaussian Noise}$$

($\boldsymbol{x}$: Clean Image; $\boldsymbol{k}$: Blurring Kernel)

- Assuming known $\boldsymbol{k}$ & noise level, estimate $\boldsymbol{x}$ from $\boldsymbol{y}$
- An ill-posed problem, requiring priors on $\boldsymbol{x}$

- Challenges
  - Find good image priors
  - Develop efficient numerical solutions

Our Proposal

- A New Parametric Image Prior
  - Captures heavy-tailed statistics of gradient distributions
  - Handles other band-pass filter responses
  - Fitting ability comparable to hyper-Laplacian

- A Fast & Effective Image Restoration Method
  - Closed-form numerical solutions
  - State-of-the-art image restoration quality & processing speed

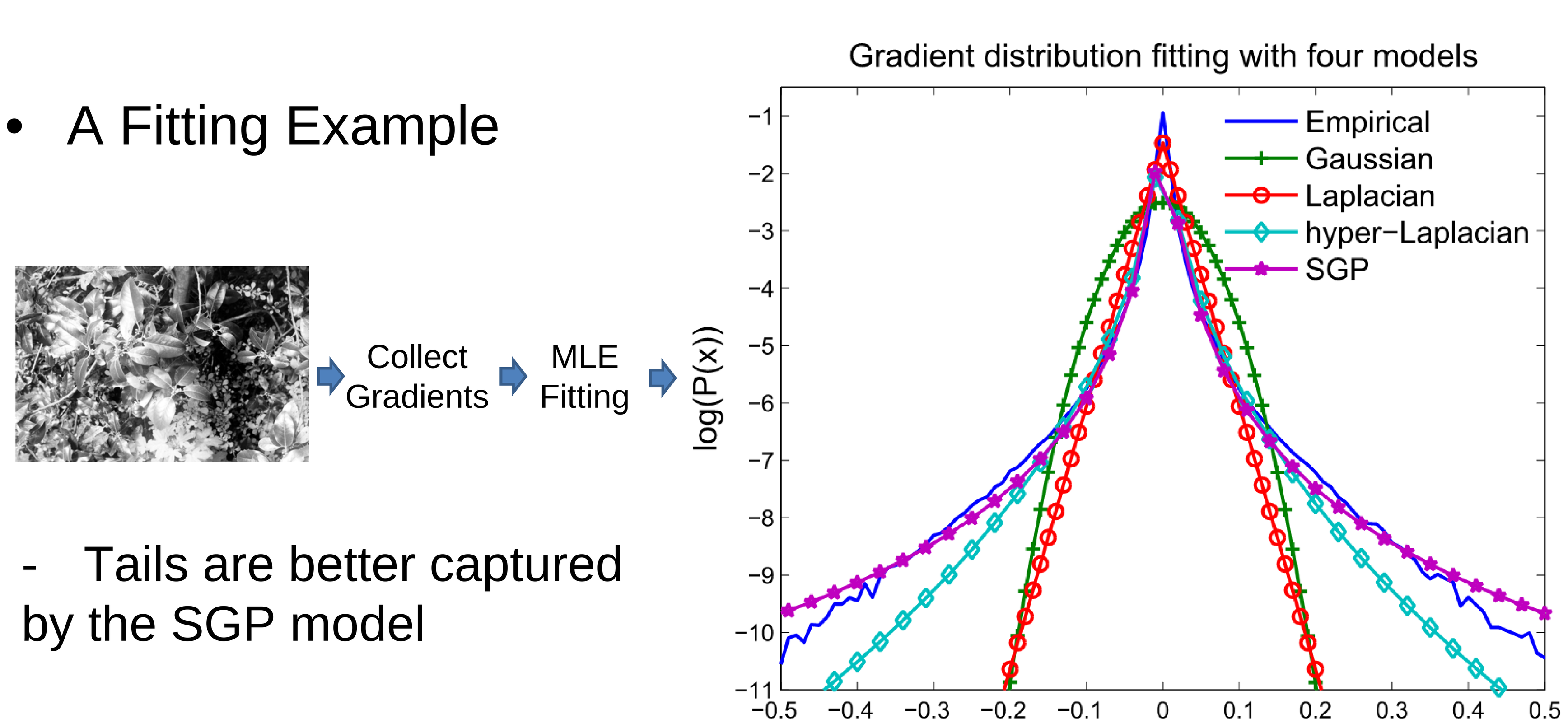
The SGP Prior

- Symmetric Generalized Pareto (SGP)

$$p(x \mid \omega, \gamma) = \frac{\omega \gamma^{\omega}}{2(|x| + \gamma)^{\omega+1}}, x \in \mathbb{R}$$

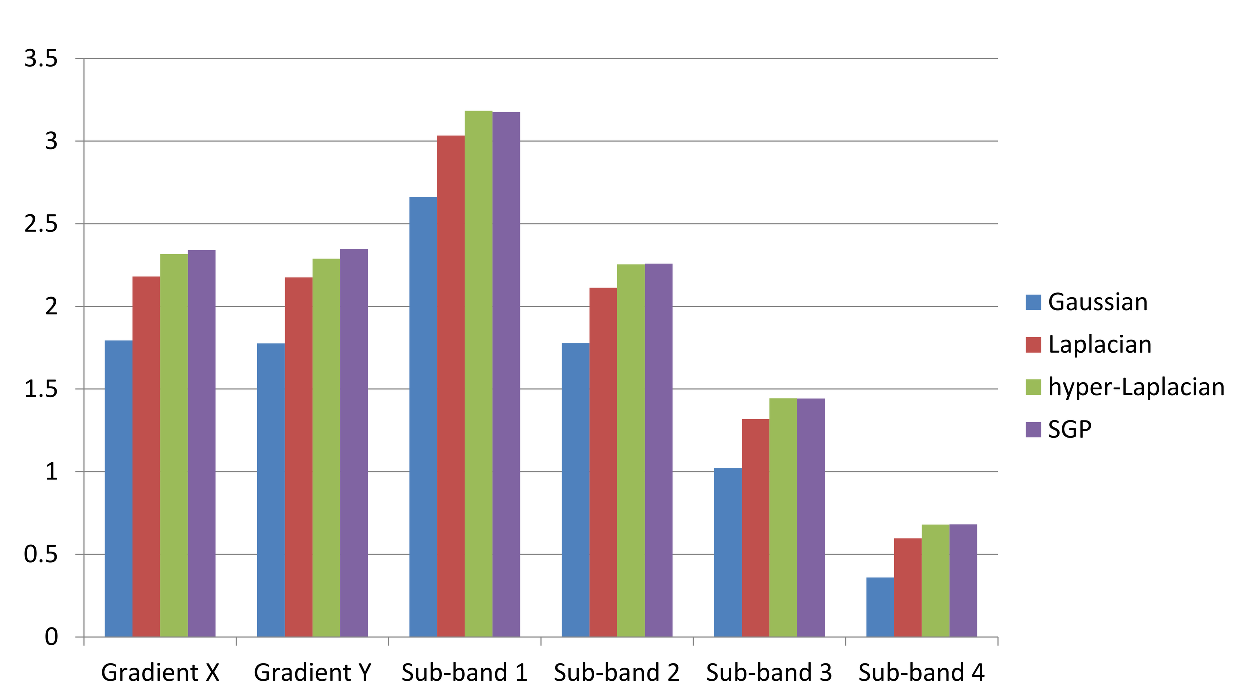
- Symmetrizes generalized pareto to the whole real line
- Tail heaviness is controlled by $\omega, \gamma > 0$

- A Fitting Example



- Tails are better captured by the SGP model

- Quantitative Evaluation
  - 100 images from van Hateren's data set
  - 6 band-pass filter responses
  - Average likelihood scores
  - SGP comparable to hyper-Laplacian



Restoration with SGP

- The Maximum-a-posterior (MAP) Formulation

$$\min_{\boldsymbol{x}} \sum_i \left(\underbrace{\frac{\lambda}{2}(\boldsymbol{y} - \boldsymbol{x} \otimes \boldsymbol{k})_i^2}_{\text{Data likelihood}} + \underbrace{\sum_{j=1}^{2} \log(|\boldsymbol{x} \otimes \boldsymbol{f}_j|_i + \gamma)}_{\text{SGP-based regularizers}} \right)$$

i - pixel index, j - band-pass (gradient) filter type

- Difficult to solve due to the non-differentiable regularizers

- Half-Quadratic Splitting Solution

- Decouples $\boldsymbol{x} \otimes \boldsymbol{f}_1, \boldsymbol{x} \otimes \boldsymbol{f}_2$ from SGP regularizers using auxiliary variables $\boldsymbol{z}_1, \boldsymbol{z}_2$ and quadratic penalty terms

$$\min_{\boldsymbol{x}, \boldsymbol{z}_1, \boldsymbol{z}_2} \sum_i \left(\frac{\lambda}{2}(\boldsymbol{y} - \boldsymbol{x} \otimes \boldsymbol{k})_i^2 + \underbrace{\sum_{j=1}^{2} \frac{\beta}{2}(\boldsymbol{x} \otimes \boldsymbol{f}_j - \boldsymbol{z}_j)_i^2}_{\text{Quadratic penalty terms, } \beta \to \infty} + \sum_{j=1}^{2} \log(|\boldsymbol{z}_j|_i + \gamma) \right)$$

- Solves two sub-problems using block coordinate descent
  - Fixed $\boldsymbol{z}_1, \boldsymbol{z}_2$, solve $\boldsymbol{x}$ with 2D FFTs and IFFTs
  - Fixed $\boldsymbol{x}$, solve $\boldsymbol{z}_1, \boldsymbol{z}_2$ in a common 1D form independently on each pixel

$$\min_z g(z) = \frac{\beta}{2}(z - v)^2 + \log(|z| + \gamma)$$

Note that when $z > 0$, $g'(z) = 0 \Rightarrow \beta(z - v)(z + \gamma) + 1 = 0$

Quadratic equations ⟶ A closed-form solution

Experimental Results

- Experimental Settings
  - 12 grayscale images, 10 blurring kernels, 3 noise levels
  - Comparison to L1 [Wang et al., SIAM JIS 2008], LUT [Krishnan and Fergus, NIPS 2009], GISA [Zuo et al., ICCV 2013]

- Quantitative Results
  - Average PSNR with 4 Gaussian kernels and 3 noise levels

Kernel	Avg. PSNR (in dB)											
	$\sigma_n^2 = 0.0001$				$\sigma_n^2 = 0.001$				$\sigma_n^2 = 0.01$			
	L1	LUT	GISA	SGP	L1	LUT	GISA	SGP	L1	LUT	GISA	SGP
13 × 13	25.87_1	25.63_4	25.64_2	25.63_3	24.95_1	24.59_4	24.61_3	24.61_2	23.57_1	23.10_4	23.12_3	23.17_2
17 × 17	24.35_1	24.07_4	24.08_3	24.08_2	23.61_1	23.28_4	23.29_3	23.32_2	22.64_1	22.26_4	22.27_3	22.33_2
21 × 21	23.25_1	23.04_4	23.05_3	23.07_2	22.74_1	22.50_4	22.51_3	22.53_2	22.04_1	21.67_4	21.69_3	21.75_2
25 × 25	22.58_1	22.40_4	22.41_3	22.42_2	22.17_1	21.93_4	21.94_3	21.97_2	21.54_1	21.19_4	21.20_3	21.27_2

$$\mathrm{L1} > \mathrm{SGP} > \mathrm{LUT} \approx \mathrm{GISA}$$

  - Average PSNR with 6 motion kernels and 3 noise levels

Kernel	Avg. PSNR (in dB)											
	$\sigma_n^2 = 0.0001$				$\sigma_n^2 = 0.001$				$\sigma_n^2 = 0.01$			
	L1	LUT	GISA	SGP	L1	LUT	GISA	SGP	L1	LUT	GISA	SGP
15 × 15	28.62_4	30.01_2	29.99_3	30.01_1	25.17_4	27.01_2	26.99_3	27.04_1	23.25_4	24.60_3	24.62_2	24.64_1
17 × 17	27.82_4	29.22_1	29.21_2	29.15_3	23.87_4	25.82_2	25.79_3	25.84_1	21.68_4	23.46_2	23.45_3	23.54_1
19 × 19	28.22_4	29.57_1	29.55_2	29.47_3	23.75_4	25.85_2	25.80_3	25.90_1	21.04_4	23.44_2	23.38_3	23.64_1
21 × 21	28.89_4	30.40_2	30.36_3	30.42_1	24.86_4	27.01_2	26.96_3	27.08_1	21.87_4	24.07_2	24.04_3	24.19_1
23 × 23	28.48_4	29.52_2	29.50_3	29.53_1	25.35_4	26.81_1	26.80_3	26.80_2	22.37_4	23.62_3	23.64_2	23.66_1
27 × 27	26.26_4	27.53_1	27.51_2	27.48_3	23.32_4	25.05_2	25.03_3	25.06_1	21.29_4	22.63_3	22.64_2	22.71_1

$$\mathrm{SGP} > \mathrm{LUT} \approx \mathrm{GISA} > \mathrm{L1}$$

- Visual Comparison
  - The 'Barbara' example: 27x27 motion kernel + 10% noise

- Running Time (in Seconds)
  - The z-step: L1, SGP - 0.014 , LUT - 0.048, GISA - 0.023
  - The x-step: 0.576

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