1. Construct deterministic finite automata (DFAs) recognizing the following languages over the alphabet \( \{a, b\} \):

(a) \( \{w \mid \text{aa is a substring of } w, \text{ but } ab \text{ is not}\} \)

(For instance, \( baa \) is in this language, but \( aab \) is not.)
(b) \( \{a^i b^j \mid i \geq 0, \ j \geq 0, \ i + j \text{ is an even number} \} \)
(c) The set of all strings that begin with $a$ but do not contain $aab$ as a substring.

(d) $\{ab\}^* \cup \{a\}$
2. Disprove the following: for all languages $A$, $B$

$$A \subseteq B \rightarrow A^* \subseteq B^*$$

(In other words, exhibit languages $A$ and $B$ such that $A$ is a proper subset of $B$, but $A^*$ is not a proper subset of $B^*$.)

$$A = \{a\}, \ B = \{\epsilon, a\}$$

3. Disprove the following: if $A \cap B = C$ then $A^* \cap B^* = C^*$.

(In other words, exhibit languages $A$, $B$, $C$ such that $A \cap B = C$ but $A^* \cap B^* \neq C^*$.)

Take $A = \{a\}, \ B = \{aa\}, \ C = \emptyset$, so $C^* = \{\epsilon\}$. But $A^* \cap B^* = \{aa\}^*$. 

(e) $\{b\} \cap \{aa\}^*$