1. Construct deterministic finite automata (DFAs) recognizing the following languages over the alphabet \{a, b\}:

(a) \{w \mid aa \text{ is a substring of } w, \text{ but } ab \text{ is not}\}

(For instance, baa is in this language, but aab is not.)

(b) \{a^i b^j \mid i \geq 0, \ j \geq 0, \ i + j \text{ is an even number}\}

(0 is an even number, so \epsilon \text{ is in this language.})

(c) The set of all strings that begin with a but do not contain aab as a substring.

(d) \{ab\}^* \cup \{a\}

(e) \{b\} \circ \{aa\}^*

2. Prove that if \(L\) is a nonempty language, then \(L \subseteq L^2\) if and only if \(\epsilon \in L\).

3. Disprove the following: if \(A \cap B = C\) then \(A^* \cap B^* = C^*\).
   (In other words, exhibit languages \(A, B, C\) such that \(A \cap B = C\) but \(A^* \cap B^* \neq C^*\).)