is a minimally connected graph; equivalently, a connected graph with no circuits.

In the course, we cover only \textbf{rooted} trees. In Graph Theory, a (non-rooted) tree

Trees

CSI 310: Lecture 21
An expression:

Each arc expresses the structural relation between the root node and the subtree.

(b) Under the root of each of the trees specified

(c) One arc from this tree's root to common with each other on the root. Its subtrees, with no nodes or arcs in zero or more rooted trees called

(d) One root node. (and)

(e) Any operator and operands under

( and) operand(s).

(f) No more expressions as operands

(g) If it has an operator, it has one.

(h) Either is an identifier or constant.

(i) Are substrings of the expression.
root.

and keep moving upward to each node’s parent, you will eventually reach the

one), then move again to that node’s parent’s parent (provided there is one),

If you start at any node and move to the node’s parent (provided there is

Each node except for the root has exactly one parent; the root has no parent.

say that “d” is c’s parent.

its children. If a node c is the child of another node d, then we

Each node may be [is] associated with [zero] or more different nodes, called

Each is one special node, called the root.

1. There is one special node, called the root:

empty, then it [the set of nodes] must follow these rules:

called the empty tree). But [logically, “not” means “and”] [if the set is not

A tree is a finite set of “nodes”. The set might be empty (no nodes, which is

Main/Secondary Definition:

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{ }
    
    node temp;
    count = count + 1;
    
    while (temp is not the root)
    
        if (temp is the root)
            
            temp = parent(temp);

    
    if (find-root(n) is n)

    
    
    The following algorithm, started at any node, always halts, eventually:

    A more formal description of Property 4:
The no-cycle property of trees. For every node in the find-root(n) algorithm

```c

    if (n is the root) 
    { 
        return node
    }
    else
    {
        return find-root(parent(n) in )
    }
```

halts when it is started on n.

Recursive root-finder
The purpose of clause 4 of the definition and not clause 4, relationship which satisfies clauses 1–3 (This is an example of a parent–child parent of 1, and 3 is the parent of 4. parent of 3, I is the parent of 2, 4 is the like nodes 3, 2, I, 4, 3, etc. Where 2 is the definition of a tree is to exclude situations clause 4, in the unique root is 5.)
is c’s parent.

Left child and its right child. If a node c is the child of node p, we say: “d, each node may be associated with up to two other different nodes, called its

Replace M/S tree definition clause (2) by:

(Binay Tree) In brief, each node’s children are distinguished from one another as

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Complete Binary Tree: Every non-leaf has exactly 2 children.

Full Binary Tree: Every leaf has the same depth.

Ullman, Corinman, Leiserson, Rivest, and others) call this the height.

Depth of a whole tree: Maximum depth of any of its leaves.

Better authors, writing for more sophisticated readers (like Knuth, Aho, Hopcroft,

Left and right subtrees of a node.

Subtree (we can view any non-root node as the root of a new, smaller tree,

Parent, Sibling, Ancestor, Descendant.

No children.

Computer Sci, examinations and job placement interviews): Leaf: A node with
Recursive algorithm to compute the depth of a node:

```c
    { else return 1 + depth(paren1) n; }

    if (n is the root) return 0;

    int depth(node n) {
        return depth(n); // Recurive call
    }
```
You can simplify this by omitting the if test:

```c
{ }
{ }
return maxer;
{ maxer = max(maxer, height(c)); }
for each of the children nodes c of n
{ int maxer = 0; }
else
{ if (n is a leaf) return 0; }
( int height(node n) )
```

Recursive algorithm to compute the height of the subtree with the given root:
Example of an expression and its Parse Tree

A = (B = (C - (D * (E++)) + (F * G)))

( = )

( top level operator is assignment )

( )

( (D * (E++)) + (F * G))

( + )

( top level operator is addition )

( (C - (D * (E++)) + (F * G))

( - )

( top level operator is subtraction )

( (C - (D * (E++)) + (F * G))

( * )

( top level operator is mult. )

( (D * (E++)) + (F * G))

( + )

( top level operator is addition )

Details cont’d on the next 2 frames

Identifier

Identification

Identifier

Identification

Identifier

Identification

Details cont’d on the next 2 frames
top level operator is multiplication ( \( \times \) )
\[
(C-(D*(E++)))
\]

*top level operator is subtraction (  )

( + )

*top level operator is multiplication ( )

\[
(C-(D*(E++)))
\]

*top level operator is subtraction (  )

( )

*top level operator is increment (  )

D

( )

identifier

C

identifier

E

identifier

D

identifier

identifier

identifier

identifier

identifier

identifier

identifier
The following C/C++ expression has an expression tree that is binary but not complete binary.
Binary Search Trees: A decision tree for answering whether or not a given question is true or false, by using a finite set of yes-no questions.

Other Name Space Trees: E.g., the Domain Name System of the Internet.

File Name Trees: Express a system to identify files using a sequence of directory names, plus a file name.

Expression Trees: Express the structure of the computation expressed by an expression (string, web document, program, etc.).

Taxonomy Trees: See http://www.ncl.dunelm.gla.ac.uk/taxonomy/ and http://www.ispl.org/ncl/faculty/steve/taxonomy/
For the tree and each subtree, the root contains the largest of the numbers in:

Heap ordered tree (\texttt{Heap}): A tree (of numbers) with the heap property.
Worst case. Use an amount of computer work proportional to the height of the tree, in the
insertion into a heap.

Search and insertion in a binary search tree,
Main conclusion:
number into a heap ordered tree.

We then viewed Main/Section's page 10 on the dictionary abstract data type and the
example of how a dictionary of states (of the USA) is implemented by a binary
search and insert a new number into it.

We viewed Main/Section's page 498 on a binary search tree and discussed how to
}